Induced Emission of Radiation from a Large Space-Station-Like Structure in the Ionosphere

D. E. Hastings* and J. Wang†
Massachusetts Institute of Technology, Cambridge, Massachusetts

Large conducting structures in the ionosphere may have currents flowing through them which close in the ionospheric plasma. These currents can arise either from current leakage from an onboard power distribution system or by being induced by the motional electric field. Associated with these currents will be broadband electromagnetic radiation in the Alfven and lower hybrid bands. The radiation impedance of this electromagnetic radiation is explored for a structure of space-station-like dimensions as a function of the geometry of the structure and the composition of the ionic environment. It is shown that modification of the collecting area of the structure and environment can be used to minimize the radiation impedance. For a space station, the radiated power will at most be of the order of watts, which does not represent a significant power loss. However, the radiation field will give rise to a substantial pollution of the electromagnetic spectrum in the vicinity of the space station. Design choices to minimize this interference are suggested.

I. Introduction

ANY conducting structure moving through the magnetic field of the Earth will feel a motional electric field and may have a dc current flowing through it in response to the field. For large conducting structures in the ionosphere, the induced potential can significantly exceed the potential associated with collection of thermal currents. For an active system such as a space station, there are two possible modifications to this picture of a dc current induced in the structure by the motion. First, there may be current leakage from the power system of the station. For a station distributing 1 kA of current among its electrical loads, then a leakage of only one part in 10^5 will give 10 mA flowing through the structure. This leakage current will add to the motionally induced current. Second, since the power system of the station will distribute alternating current, there may be inductive coupling between the power distribution system and the structure current. Hence, for a large active system such as a space station, we may expect that possibly an alternating current (of the order of tens of milliamperes) may flow through the structure and close in the ionosphere. Since this current source will be moving across the geomagnetic field, there will be electromagnetic radiation emitted from the moving structure. With the reasonable assumption that the response of the plasma to the current source is linear, it is possible to assign a radiation impedance to the current source such that the radiated power is the square of the current in the structure multiplied by the radiation impedance.

The radiation impedance of conducting objects in space has been the subject of several investigations.[1-4] The seminal work on this subject was by Drell et al.,[3] who described the excitation and radiation of Alfven waves from moving bodies in magnetoplasmas. It was in this work that the concept of Alfven wings was first introduced. It was shown that Alfven waves radiated from the body could carry energy away from the body, thus leading to a drag force on the body. In the work of Belcastro et al.,[2] and Rasmussen et al.,[4] the radiation impedance was computed for radiation of Alfven waves from a tether-like object. This impedance was shown to be low, of the order of a few Ohms. These works concentrated on the impedance in the Alfven regime because of the belief that the dimensions of the tether system were such that radiation of small wavelength waves would not naturally occur. Hence, the radiation impedance associated with waves above the ion cyclotron frequency would be vanishingly small. This assumption was examined in the work of Barnett and Olbert,[1] where the radiation impe-dance was computed both for radiation of Alfven waves and for the radiation of waves in the lower hybrid range of frequencies. It was shown there that the excitation of lower hybrid waves in the plasma could make a substantial contribution to the radiation impedance of a tethered system. However, the model of a tether that they took was unrealistic since the tether was modeled as a thin cylinder with no end connectors. In recent work by Hastings and Wang,[5] a tether was modeled with end conductors, and it was shown that the radiation impedance was substantially lowered by the presence of the end connectors. This suggests the importance of geometry to the radiated power from a space structure.

In this work we shall examine a structure of space-station-like dimensions which carries an oscillating current of a given magnitude. This current flows in and out of the structure through the conducting surfaces of the solar arrays and the main structure and closes in the ionosphere. The magnitude of the current will depend on many other factors in addition to the radiation impedance, including the contact impedance and the Ohmic resistance as well as the amount of leakage from the power distribution system. Since the electric and magnetic wave fields will scale linearly with the current magnitude, we just choose a given current rather than undertake the current calculation for some specified cases.

The basic radiation theory for a large conducting structure in a single-ion plasma is worked out in detail in Ref. 6. In this work we extend the analysis of Ref. 6 to a multiple-ion plasma and systematically consider the effects of geometry and ionic composition on the radiation impedance. From these radiation impedances, we estimate the average magnitudes of the electric and magnetic fields per unit current. However, given the complexity of calculating the currents, we consider the major results of this work to be the radiation impedance for a space

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*Charles Stark Draper Assistant Professor of Aeronautics and Astronautics, Department of Aeronautics and Astronautics. Member AIAA.

†Research Assistant, Department of Aeronautics and Astronautics.

station structure and its dependence on the environmental and structural parameters. Since our theoretical formulation is only valid in the far field of the structure, the electric and magnetic fields calculated will be lower bounds on the measurements from such a structure. This work is motivated by the realization that radiation of electromagnetic waves from a structure such as the space station may have a negative impact on measurements made from the structure. Therefore, understanding of the radiation and possible alleviation will suggest design choices for large structures such as the space station.

II. Formulation of the Radiated Power from a Conducting Space Structure

We begin our formulation in the rest frame of the ambient plasma. The mechanism of the radiation by a moving conductor in a magnetized plasma is analogous to Cherenkov radiation. It is easy to show that of all the waves a plasma can support, the ones that will be radiated by the moving conductor will be the ones that satisfy the Doppler condition.[2]

$$\omega = \pm \omega^* + \boldsymbol{k} \cdot \boldsymbol{V} \tag{1}$$

where ω^* is the ac frequency of the current in the structure. For a cold single-ion species plasma, the radiated plasma waves will occur in two main bands. (There is a third band, but the inclusion of warm plasma effects will eliminate this band.) The first is the Alfven band, $0 < \omega < \Omega_i$, where Ω_i is the ion cyclotron frequency. The radiation in this band is spread over the whole band[6] and has been extensively investigated in works that consider the MHD response of the plasma. The second band is the lower hybrid band $\omega_{lh} < \omega < \Omega_e$, where ω_{lh} is the lower hybrid frequency $\omega_{lh} \simeq \sqrt{\Omega_i \Omega_e}$, and Ω_e is the electron cyclotron frequency. The radiation in this band is mainly concentrated around ω_{lh}.

We shall consider the space structure as a generalized box, as shown in Fig. 1. The structure is taken to be moving perpendicular to the magnetic field in the x_1 direction while the field is in the x_3 direction. The structure is oriented along the x_2 direction, which is the $\boldsymbol{V} \times \boldsymbol{B}$ direction.

The time-averaged radiated power from a moving conductor in a cold plasma carrying an alternating source current of frequency ω^* consists of power radiated at ω^* due to line radiation and continuum power radiated because the conductor is moving. We concentrate on the continuum radiation because it is that radiation which gives rise to the greatest electromagnetic interference, and we calculate it from classical antenna theory.[1] In addition, it can be shown that the continuum power is much larger than the line radiation and is the power that survives in the limit of vanishing ac frequency.[6] Hence, we find

$$\bar{P}_{\text{rad}} = \sum_{i=1}^{N+1} \int_{\text{band } i} \text{d}\omega \, \frac{\text{d}\bar{P}_{\text{rad}}}{\text{d}\omega} \tag{2}$$

where the differential power is given by

$$\frac{\text{d}\bar{P}_{\text{rad}}}{\text{d}\omega} = \frac{\pi^2}{cV} \int_{-\infty}^{\infty} \text{d}k_2 \left[\left(\frac{|\boldsymbol{j}_s \cdot \boldsymbol{k}|^2}{k_\perp^2} \frac{1}{\sqrt{\beta S}} \right)_{k_1 = (\omega - \omega^*)/V} + \left(\frac{|\boldsymbol{j}_s \cdot \boldsymbol{k}|^2}{k_\perp^2} \frac{1}{\sqrt{\beta S}} \right)_{k_1 = (\omega + \omega^*)/V} \right] \tag{3}$$

where

$$\beta = \frac{\omega^2 P(\omega)}{\omega^2 P(\omega) - c^2 k_\perp^2} \tag{4}$$

In Eq. (3) the perpendicular wave number is $k_\perp$, given from $k_\perp^2 = k_1^2 + k_2^2$, where k_1, k_2 are the components of the wave vector in the x_1 and x_2 directions, and the functions $P(\omega)$ and

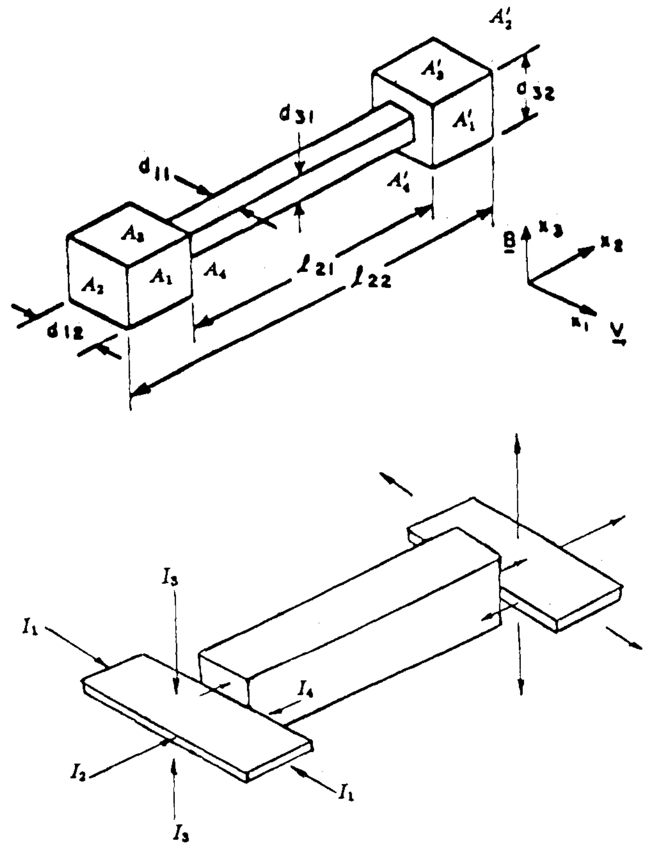

Fig. 1 Generalized box structure for source current.

$S(\omega)$ are the diagonal parallel and perpendicular elements of the cold plasma dielectric tensor. The radiation bands come from the requirement that $S(\omega) > 0$. For a single-ion species plasma, the radiation bands can be determined as $0 - \Omega_i$, $\omega_{lh} - \Omega_e$. For a multi-ion species plasma (with N ion species), we solve the equation

$$S(\omega) = 1 - \frac{\omega_{pe}^2}{\Omega_e^2 - \omega^2} + \sum_{k=1}^{N} \frac{\omega_{pik}^2}{\Omega_{ik}^2 - \omega^2} = 0 \tag{5}$$

numerically, and find that there are $N+1$ radiation bands given by $0 - \Omega_{i1}$, $\omega_{lh1} - \Omega_{i2}$, $\omega_{lh2} - \Omega_{i3}, \ldots \omega_{lhN} - \Omega_e$. In Eq. (5), ω_{pe} is the electron plasma frequency, Ω_{ik} is the ion cyclotron frequency of the kth ion species, ω_{pik} is the ion plasma frequency of the kth ion species, and ω_{lhk} is the kth root of $S(\omega) = 0$.

We can estimate the power radiated by integrating over the bands. In the results in this paper, the integral was performed numerically. A Gaussian 64-point integration was used for the ω frequency integral, and the Fourier cosine integrals arising from the k_2 integration were performed by a special quadrature package designed for highly oscillatory integrands. The impact of warm plasma effects was indicated by cutting off the k_2 integral at $(k_\perp \rho_e)^2 > 1$. We cut off the k_2 merely to indicate when the warm plasma effects become important. We find that this cutoff significantly affects the values of the integrals for $\omega^* > 10^5$ Hz. Hence, above this frequency our cold plasma results are not to be believed. For lower ac frequencies, our results for the radiation are lower bounds. This is true for two reasons. First, the formulation for the radiated power is strictly valid only in the far field, so that the near-field electric fields are likely to be larger than observed here. Secondly, warm plasma effects allow the source to radiate many plasma waves (for example, electron plasma waves) that are then absorbed in the near-field plasma. Hence, the far-field cold plasma radiation is the minimum amount of radiation. Numerical calculations were performed for different geometry structures and different environments.

The velocity of the source is $V = |V|$, and $|j_s \cdot k|$ is the Fourier transformed normal source current density at the surface of the structure in a frame moving with the structure. The source current density in real space as a function of space and time is

$$J_s(x,t) = \begin{cases} J_s(x)\cos(w^* t) & x \text{ inside surface } F(x') = 0 \\ 0 & x \text{ elsewhere} \end{cases} \tag{6a}$$

where x' are the coordinates in the frame moving with the conductor:

$$x' = x - Vt \tag{6b}$$

and the conductor is bounded by $F(x') = 0$.

This form contains the information that the current is alternating (through ω^*) and is moving with the body. The Fourier transform of J_s is

$$J_s(k,\omega) = \sqrt{2\pi} j_s(k) \tfrac{1}{2} [\delta(\omega + \omega^* - k \cdot V) + \delta(\omega - \omega^* - k \cdot V)] \tag{7}$$

where $j_s(k)$ is the transform of $J_s(x)$.

For the space-station-like structure in Fig. 1, we find that the general expression for the normal surface current is

$$|j_s \cdot k| = \frac{2}{(2\pi)^{3/2}} \left\{ \frac{I_2}{A_2}\left[(d_{12}d_{32}) \frac{\sin(k_1 d_{12}/2)}{k_1 d_{12}/2} \frac{\sin(k_3 d_{32}/2)}{k_3 d_{32}/2} \times \sin\left(\frac{k_2 l_{22}}{2}\right)\right] + \frac{I_4}{A_4}\left[(d_{12}d_{32}) \frac{\sin(k_1 d_{12}/2)}{k_1 d_{12}/2} \frac{\sin(k_3 d_{32}/2)}{k_3 d_{32}/2} - (d_{11}d_{31}) \frac{\sin(k_1 d_{11}/2)}{k_1 d_{11}/2} \frac{\sin(k_3 d_{31}/2)}{k_3 d_{31}/2} \right] \sin\left(\frac{k_2 l_{21}}{2}\right) \right]$$
$$+ \frac{I_1}{A_1}\left[2d_{32} \frac{(l_{22} - l_{21})}{2} \right] \left[\frac{\sin[k_2(l_{22} - l_{21})/4]}{k_2(l_{22} - l_{21})/4} \right] \times \cos\left(\frac{k_1 d_{12}}{2}\right) \frac{\sin(k_3 d_{32}/2)}{k_3 d_{32}/2} \sin\left[\frac{k_2(l_{22} + l_{21})}{4}\right]$$
$$+ \frac{I_3}{A_3}\left[2d_{12} \frac{(l_{22} - l_{21})}{2} \right] \left[\frac{\sin[k_2(l_{22} - l_{21})/4]}{k_2(l_{22} - l_{21})/4} \right] \times \cos\left(\frac{k_3 d_{32}}{2}\right) \frac{\sin(k_1 d_{12}/2)}{k_1 d_{12}/2} \sin\left[\frac{k_2(l_{22} + l_{21})}{4}\right] \Bigg\} \tag{8}$$

The power in Eq. (2) is the asymptotic power that arrives at distances far from the source and is computed from the Poynting flux in the far field propagating through a plane perpendicular to the magnetic field.

We define a radiation impedance for each band as

$$Z_{\text{band } i} = \left(\int_{\text{band } i} d\omega \frac{d\bar{P}_{\text{rad}}}{d\omega} \right) \Big/ I^2 \tag{9}$$

where the average current flowing in the structure is I. The total impedance is

$$Z_{\text{rad}} = \sum_{i=1}^{N+1} Z_{\text{band } i} \tag{10}$$

Finally, the radiated power is given by

$$\bar{P}_{\text{rad}} = I^2 Z_{\text{rad}} \tag{11}$$

III. Radiation from a Space-Station-Like Structure

Radiation in Single-Ion Plasma

The plasma environment at LEO is composed mainly of O^+ ions. Hence, we shall study the radiation in an O^+ plasma with typical LEO parameters: magnetic field $B = 0.33$ Gauss; electron and ion number density $n_e = n_i = 2 \times 10^5$ cm^{-3}; electron temperature $T_e = 0.1$ eV. These parameters are typical of the dayside environment at 300 km. The space structure is taken to be moving with a velocity $V = 7.3$ km/s. In such an environment, the wave radiation bands are found to be as follows: band I, $0 < \omega < \Omega_i = 31.3$ Hz; and band II, $\omega_{lh} = 5.4 \times 10^3 < \omega < \Omega_e = 9.2 \times 10^5$ Hz. A practical space station with solar array can be simulated by our box structure with dimensions $l_{21} = 80$ m, $l_{22} = 100$ m, $d_{11} = d_{31} = 5$ m, $d_{12} = 20$ m, $d_{32} = 0.5$ m, and the current is assumed to flow through the solar arrays and end surfaces of the main body, i.e., surfaces A_1, A_2, A_3, and A_4 and A_1', A_2', A_3', and A_4', where A_i are all on the left box, and A_i' are all on the right box in Fig. 1. The normal current densities at each surface are uniform and are assumed the same.

For comparison, the impedance of a space station without solar arrays is also calculated. This structure can be easily simplified from the generalized box with dimensions taken to be $l_{21} = l_{22} = 100$ m, $d_{11} = d_{31} = d_{12} = d_{32} = 5$ m and current flowing in and out of the system only through surfaces $A_2 - A_2'$.

The impedances at some typical ac frequencies are shown in Table 1. We see that the impedances of both structures due to band I decay quickly when $\omega^* > 10^2$ Hz, whereas the impedances in band II jump to a peak value around $\omega^* \sim 10^4$ Hz. The results show that there is great difference between the impedance of a station with and without large conducting arrays. This will be discussed in the next section.

From the radiation impedance, we can estimate the average fluctuation magnitudes of the electric and magnetic field. In our problem the wave packets propagate close to or along the lines of the magnetic field.[1] Detailed calculation shows that most of the radiation is in lower hybrid waves over most of the ac frequency range, and the power radiation spectrum is concentrated near ω_{lh}. Hence, the radiated energy propagates along the magnetic field with the group velocity U_g of the lower hybrid waves. The average magnitudes of the electric and magnetic field approximately satisfy

$$P_{\text{rad}} \simeq \tfrac{1}{2}[(1/\mu)\langle \delta B \rangle^2 + \epsilon \langle \delta E \rangle^2] U_g A \tag{12}$$

where A is the projected area of the structure along the external magnetic field. Finally, the average electric and magnetic field fluctuations in the far field are given by

$$\langle \delta E \rangle \simeq U_{\text{ph}} \left(\frac{\mu_0 P_{\text{rad}}}{U_g A} \right)^{1/2} = U_{\text{ph}} \left(\frac{\mu_0 Z_{\text{rad}}}{U_g A} \right)^{1/2} I \tag{13}$$

$$\langle \delta B \rangle \simeq \left(\frac{\mu_0 P_{\text{rad}}}{U_g A} \right)^{1/2} = \left(\frac{\mu_0 Z_{\text{rad}}}{U_g A} \right)^{1/2} I \tag{14}$$

Table 1 Impedances of space station (Ω)

	Station with solar array		Station without solar array	
ω^*, Hz	Band I	Band II	Band I	Band II
1	5.52×10^{-2}	3.07×10^{-2}	9.23×10^{-2}	2.32
100	1.12×10^{-2}	2.52×10^{-2}	4.69×10^{-2}	2.38
400	2.12×10^{-4}	3.29×10^{-2}	2.78×10^{-2}	2.96
10^3	2.14×10^{-5}	2.73×10^{-2}	4.49×10^{-3}	2.33
10^4	1.28×10^{-7}	2.17	6.60×10^{-6}	2.12×10^2
2×10^4	4.46×10^{-8}	6.49×10^{-2}	3.53×10^{-6}	1.12×10^1

where U_{ph} is the phase velocity of lower hybrid waves. The values of $\langle \delta E \rangle$ and $\langle \delta B \rangle$ per unit current are shown in Table 2. For an observer in the moving space station, the coordinate transfer, Eq. (6b), leads to $\omega' = \omega - \boldsymbol{k} \cdot \boldsymbol{V}$. Hence, instead of seeing a broadband spectrum, we shall see the field fluctuations at $\omega' = \omega^*$.

For comparison, the current specifications for the U.S. space station are as follows:

1) Electric field specifications: 3 mV/m from 20 Hz to 10 kHz, then a linear fall to 0.1 mV/m at 20 kHz, then a linear rise to, 3 V/m at 20 Ghz.

2) Magnetic field specification: 10^4 nT at 10 Hz, then a linear fall to 10^{-2} nT from 1-300 kHz.

Hence, if the structure current is approximately 10 mA and oscillates at 20 kHz, the far-field electric field will be within the specifications, but the magnetic field will considerably exceed the specifications. Since the current theory is based on the cold-plasma assumptions, the results presented here do not include the effects due to finite temperature, which will give rise to a more complicated and substantial pollution of the electromagnetic spectrum in the vicinity of the space station.

Radiation in Multiple-Ion Plasma

Although in the LEO environment the dominant ion species is O^+ and the number densities of other ion species are very small, there are reasons for us to consider the environment as a multi-ion species plasma. The first reason is that, as long as there is a second ion species appearing, no matter how small the number density might be, a new radiation frequency band will open, and the entire radiation behavior will be modified. The second is that the surrounding environment can be changed significantly due to the contamination from the space structure. For example, the Space Shuttle is observed to have a cloud of water plasma around it.[7] Third, above 400 km the LEO environment varies at night from mainly O^+ to O^+ combined with He^+ and H^+. In this paper, for simplicity, we shall consider only a two-ion species plasma. We consider a plasma composed of water ions and oxygen ions. The densities are taken to be

$$n_{H_2O^+}/n_e = 0.9, \qquad n_{O^+}/n_e = 0.1, \qquad n_e = 2 \times 10^5 \text{ cm}^{-3}$$

Instead of two bands, there are three radiation bands because of the appearance of the second ion species. The radiation frequency bands for this environment are as follows: band I, $0<\omega<\Omega_{iH_2O^+}$; band II, $\omega_{lh1}<\omega<\Omega_{iO^+}$; and band III, $\omega_{lh2}<\omega<\Omega_e$. Here $\Omega_{iH_2O^+} = 28$ Hz, $\omega_{lh1} = 31$ Hz, $\omega_{lh2} = 5 \times 10^3$ Hz, and Ω_{iO^+} and Ω_e are the same as before.

Once again we calculate the radiation impedance for the space station. In Tables 3 and 4 are the impedances at several typical ω^*. These results show that the behavior of the impedances in bands I and III are almost the same as that in an O^+ plasma, and the impedance associated with band II is always very small. This is not surprising. On examination of the function $S(\omega)$, we see that the effect of a multi-ion plasma comes from the mass of the ions. Since the mass of H_2O^+ ($m_{H_2O^+} = 18$ amu) is almost the same as the mass of O^+ ($m_{O^+} = 16$ amu), the environment of 90% H_2O^+ 10% O^+ is almost the same as 100% O^+ plasma for the radiation problem.

For the purpose of exploring the effect of multiple ion species, we choose two ion species with very different ion masses, namely, H^+ and O^+. The environment is taken to be

$$n_{O^+}/n_e = 0.5, \qquad n_{H^+}/n_e = 0.5$$

and other parameters are the same as before. In this environment, the radiation bands are found to be as follows: band I, $0<\omega<\Omega_{iO^+}$; band II, $\omega_{lh1}<\omega<\Omega_{iH^+}$; and band III, $\omega_{lh2}<\omega<\Omega_e$. Here $\Omega_{iH^+} = 500$ Hz, $\omega_{lh1} = 126$ Hz, and $\omega_{lh2} = 1.53 \times 10^4$ Hz. Typical impedances are given in Table 5.

Table 2 Average magnitudes of the electric and magnetic field fluctuation per unit current ($\langle \delta E \rangle V/m/A$, $\langle \delta B \rangle T/A$)

	Station with solar array		Station without solar array	
ω^*, Hz	$\langle \delta E \rangle$	$\langle \delta B \rangle$	$\langle \delta E \rangle$	$\langle \delta B \rangle$
1	7.7×10^{-4}	1.7×10^{-7}	4.1×10^{-3}	9.2×10^{-7}
100	5.0×10^{-4}	1.1×10^{-7}	4.1×10^{-3}	9.3×10^{-7}
400	4.8×10^{-4}	1.0×10^{-7}	4.5×10^{-3}	1.0×10^{-6}
10^3	4.3×10^{-4}	9.9×10^{-8}	4.0×10^{-3}	9.1×10^{-7}
10^4	3.8×10^{-3}	8.8×10^{-7}	3.8×10^{-2}	8.7×10^{-6}
2×10^4	6.7×10^{-4}	1.5×10^{-7}	8.8×10^{-3}	2.0×10^{-6}

Table 3 Impedance of space station in 90% H_2O^+-10% O^+ plasma (Ω)

	Station with solar array		
ω^*, Hz	Band I	Band II	Band III
1	4.85×10^{-2}	3.36×19^{-4}	3.67×10^{-2}
100	9.47×10^{-3}	1.23×10^{-4}	3.48×10^{-2}
400	1.80×10^{-4}	2.16×10^{-6}	2.95×10^{-2}
10^3	1.81×10^{-5}	2.22×10^{-7}	2.54×10^{-2}
10^4	1.09×10^{-7}	1.29×10^{-9}	7.66×10^{-1}
2×10^4	3.77×10^{-8}	4.46×10^{-10}	4.49×10^{-2}

Table 4 Impedance of space station in 90% H_2O^+-10% O^+ plasma (Ω): no solar arrays

	Station without solar array		
ω^*, Hz	Band I	Band II	Band III
1	8.01×10^{-2}	6.80×10^{-4}	4.46
100	3.97×10^{-2}	4.76×10^{-4}	4.33
400	2.36×10^{-2}	2.76×10^{-4}	2.81
10^3	3.81×10^{-3}	4.46×10^{-5}	1.95
10^4	5.60×10^{-6}	6.37×10^{-8}	9.95×10^1
2×10^4	3.00×10^{-6}	3.25×10^{-8}	2.07

Table 5 Impedance of space station in 50% H^+-50% O^+ plasma (Ω)

	Station with solar array		
ω^*, Hz	Band I	Band II	Band III
1	7.63×10^{-2}	2.09×10^{-1}	9.70×10^{-3}
100	1.55×10^{-2}	4.84×10^{-1}	1.12×10^{-2}
400	2.94×10^4	4.95×10^{-1}	9.28×10^{-3}
10^3	2.98×10^{-5}	4.43×10^{-3}	1.04×10^{-2}
10^4	1.78×10^{-7}	1.03×10^{-5}	2.12×10^{-2}
2×10^4	6.19×10^{-8}	4.13×10^{-6}	4.21

For brevity, we give only the results for a space station with solar array panels.

The results show that a multi-ion plasma environment such as H^+-O^+ plasma may have a significantly different radiation impedance associated with it as compared to a single-ion environment. This will be studied in next section.

IV. Factors Affecting the Radiation Impedance

From the results in the last section, we see that the radiation impedance is a strong function of the structures and environment. On examination of the radiation formulae in Fourier space,

$$Z_{\text{rad}} = Z_{\text{rad}}^{+} + Z_{\text{rad}}^{-} \tag{15a}$$

$$Z_{\text{rad}}^{\pm} = \int \left(\frac{|\boldsymbol{j}_s \cdot \boldsymbol{k}|}{I} \right)^2_{k_1 = (\omega \pm \omega^*)/V} Q \, dk_1 \, dk_2 \tag{15b}$$

we notice that, when $\omega^* = 0$, 1) plasma properties affect Z_{rad} through the radiation bands and the term $Q = \pi^2/(ck_\perp^2\sqrt{\beta S})$, and 2) the structure geometry and boundary current flow pattern affect Z_{rad} through the term $|j_s \cdot k|/I$. When $\omega^* \neq 0$, Q and $|j_s \cdot k|I$ are also functions of the ac frequency ω^*, and so ω^* affects Z_{rad} through both the $|j_s \cdot k|/I$ and the Q term.

Since the waves discussed here are excited by a moving conductor in a plasma medium, the radiation is the result of the interaction between the space structures and the environment. We shall call $|j_s \cdot k|/I$ the "structure factor" and Q together with radiation bands the "environment factor." When the whole structure-environment system is driven by $\omega^* \neq 0$, then ω^* will affect both the structure factor and environment factor. Therefore, we can always regard the radiation impedance as a function of environment, structure, and ac frequency: $Z_{rad} = Z_{rad}$ (structure, environment, ω^*).

Unfortunately, the relationship between Z_{rad} and the structure and environment factors is implicit in Eqs. (15). Even for our simplified box structure model, it is impossible to extract analytically. In order to elucidate the dependance, we shall explore the relationship numerically.

Impedance vs Structure

The structure factor includes the boundary current pattern and conductor geometry. For the purpose of exploring the functional relation, we investigate a space station with the current allowed to enter the body from some given direction. The boundary current density is taken to be uniform. In the structure, the boundary current pattern effect is simplified to that of boundary current in different directions, and the geometry effect is simplified to that of collecting surface area and direction. Each time, we shall insulate some or all of the collecting surfaces so that the boundary current only flows in the desired direction. The following cases were chosen: 1) current I_1 flows only through surface A_1 and A_1'; 2) current I_2 flows only through surface A_2 and A_2'; 3) current I_3 flows only through surface A_3 and A_3'; 4) current I_4 flows only through surface A_4 and A_4'; 5) current I_5 flows through every surface; and 6) current I_6 flows through every surface, but there are no solar arrays on the station.

The $|j_s \cdot k|$ expressions for each one of the six cases can be simplified appropriately from Eq. (8). The current collection at any surface will be similar to that known from probe theory. A sheath will form over the surface, allowing only some fraction of ions and electrons to get to the surface and be collected.

Since the length of the structure in the x_2 direction is the same, we shall assume the same total current flows in each of the above systems,

$$I_5 = I_6 = I_1 = I_2 = I_3 = I_4$$

and compare the radiation impedance. The radiation impedances are calculated in an O^+ plasma.

The impedances for space station are compared in Figs. 2 and 3. The associated electric and magnetic field fluctuations per unit current are given in Figs. 4 and 5. We find

$$Z_1 > Z_4 > Z_6 > Z_2 > Z_5 > Z_3$$

except at $\omega^* \sim 10^4$ Hz. From these results, we see that the flat solar arrays amplify the perturbation from the (A_1,A_1'), (A_2,A_2'), and (A_4,A_4') surfaces, whereas it decreases the perturbation from the (A_3,A_3') surface. Hence, it is important for a space station to have a large collecting surface facing x_3 or the magnetic field direction and to insulate other surfaces in order to have low radiation impedance. We have also examined the geometry effects for a tether with cubic end connectors and other structures. Analyzing those results, we can draw the following important conclusions:

1) The impedance increases as total collecting area decreases for any given boundary current.

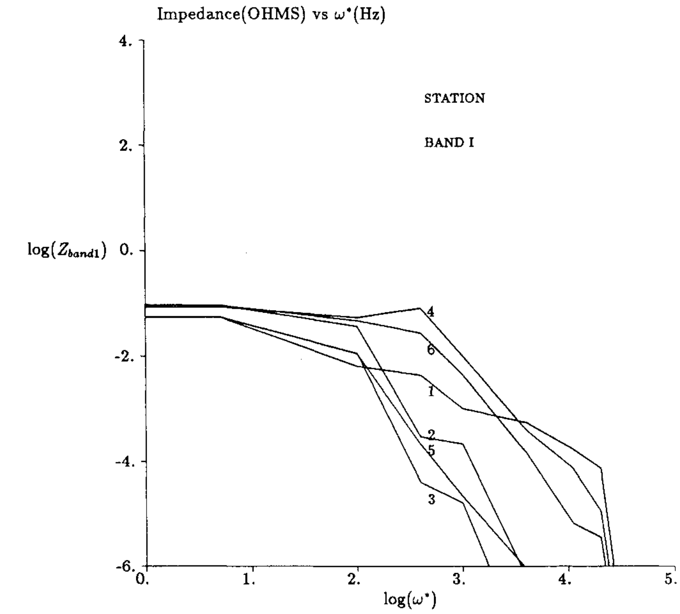

Fig. 2 Impedance in band I for the space station for different directions of current flow into the body. 1) Current I_1 flows only through surface A_1 and A_1'. 2) Current I_2 flows only through surface A_2 and A_2'. 3) Current I_3 flows only through surface A_3 and A_3'. 4) Current I_4 flows only through surface A_4 and A_4'. 5) Current I_5 flows through every surface. 6) Current I_6 flows through surface A_2 and A_2', but there are no solar arrays on the station.

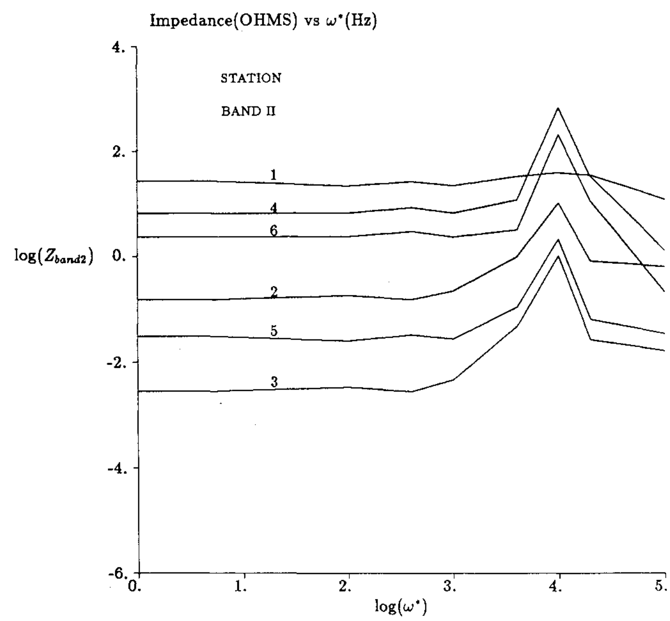

Fig. 3 Impedance in band II for the space station for the same directions of current flow into the body as in Fig. 2.

2) For a system with the same current collecting area, if the current is collected only in the x_3 direction (surface A_3, A_3'), we can get the minimum impedance; if the current is collected only in the x_1 direction (surface $A_1 - A_1'$), we get the maximum impedance. (This may not hold at $\omega^* \sim 10^4$.) If the current is collected through every surface, the impedance lies between Z_1 and Z_3.

The reasons for these conclusions can be understood as follows. Since we are assuming the same current flow in each case, increasing the collecting area means decreasing the current density at the interface; thus, the wave intensity corresponding to this current density decreases. This reduces the radiated energy, thus decreasing the radiation impedance. If the collecting areas are the same and assuming the same current density in every direction, since the plasma is a good conductor in the direction parallel to the magnetic field and a poor conductor in the perpendicular direction, it is much easier for the current to flow along the magnetic field direction than across it, and so we find the minimum resistance in this direction. For the perpendicular direction, the current conduction is poor, but since the induced field is $E' \sim V \times B$ in the x_2 direction, it is easier for the current to flow in this direction. Thus, we find $Z_3 < Z_2 < Z_1$. Since Z_5 depends on the superposition of effects of currents in every direction, we have $Z_3 < Z_5 < Z_1$. It should be noted that the above comparisons are made under the assumptions that the same total current flows for different systems and the boundary current density is the same at every direction.

Impedance vs Environment and ω^*

We have shown that impedance is a strong function of the environment from our previous results. The effect of the environment enters the formulation through the ion plasma frequencies. Since it is also clear that the effect from the environment is strongly coupled with that from the alternating frequency ω^*, we shall discuss the dependences of the impedance on the plasma environment and on the alternating frequency jointly.

For simplicity, we investigate a two-ion species plasma. The ion species are chosen to be those of very different mass. Without loss of generality, we consider a plasma composed of O^+ and H^+. We change the number density of O^+ and H^+ but always keep $n_{O^+} + n_{H^+} = n_e$. The radiation impedance will be studied as a function of n_{O^+}/n_{H^+}.

In an $H^+ - O^+$ plasma environment, the radiation bands are as follows: band I, $0 < \omega < \Omega_{iO^+}$; band II, $\omega_{lh_1} < \omega < \Omega_{iH^+}$; and band III, $\omega_{lh_2} < \omega < \Omega_e$.

Shown in Table 6 are ω_{lh_1} and ω_{lh_2} for different density ratios in a $H^+ - O^+$ plasma, as n_{H^+} increases (n_{O^+} decreases), ω_{lh_1} decreases and ω_{lh_2} increases. Thus the width of the new band $|\Omega_{iH} - \omega_{lh_1}|$ becomes larger and larger, and the width of band III $|\Omega_e - \omega_{lh_2}|$ becomes smaller and smaller. This result can be extended to an arbitrary two-ion species plasma. In a plasma originally composed of single heavy-ion species, when light-ion species are added, the original band $|\omega_{lh}^{\text{heavy}} - \Omega_e|$ will divide into two bands: $|\omega_{lh_1} - \Omega_i^{\text{light}}|$ and $|\omega_{lh_2} - \Omega_e|$. As n_{light} increases, the width of new band II will increase (but $\omega_{lh_1} > \Omega_i^{\text{heavy}}$), and the width of new band III will decrease toward Ω_e. When $n_{\text{light}} = n_e$ ($n_{\text{heavy}} = 0$), bands I and II combine to one band $|0 - \Omega_i^{\text{light}}|$, and band III becomes $|\omega_{lh}^{\text{light}} - \Omega_e|$. We note in the above process that we always have

$$\Omega^{\text{heavy}} < \omega_{lh_1} < \omega_{lh}^{\text{heavy}} < \Omega^{\text{light}} < \omega_{lh_2} < \omega_{lh}^{\text{light}} < \Omega_e$$

It is interesting to see how the radiation impedance of a space station responds to a change in the surrounding plasma. The impedances of a space station with and without solar arrays at $\omega^* = 400$ Hz and $\omega^* = 20$ kHz are shown in Figs. 6 and 7. The dimensions and current pattern are the same as before. We chose these two frequencies since they are typical frequencies for space-based power distribution systems.

From those results, we notice the following:

1) At $\omega^* = 400$ Hz, the total impedance for a space station with solar arrays is dominated by the contribution from band II and takes its maximum value for small n_{H^+}/n_e. When n_{H^+}/n_e increases, the impedance decreases.

2) AT $\omega^* = 20$ kHz, in contrast to the previous case, the total impedance is dominated by the contribution from band III. This contribution has a minimum value for $n_{H^+}/n_e \sim 2/3$. We also find that for the space station we always have

$$Z^{H^+ - O^+} \gg Z^{H^+} > Z^{O^+}$$

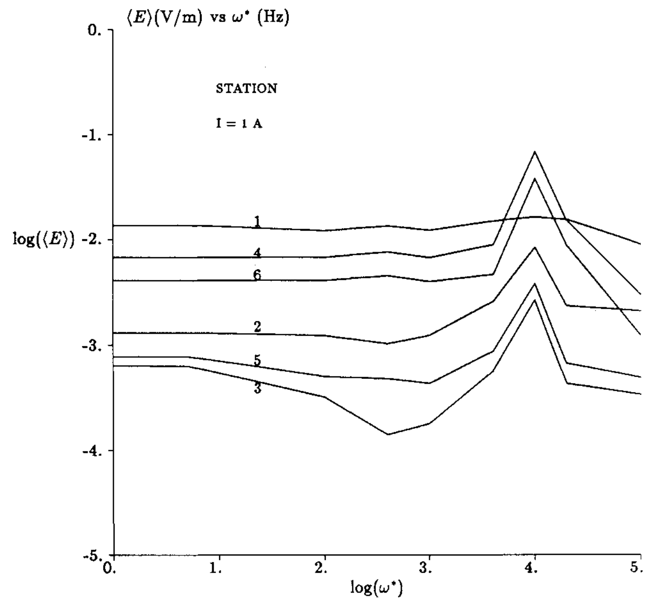

Fig. 4 Average magnitude of the electric field fluctuation per unit current for the same directions of current flow as in Fig. 2.

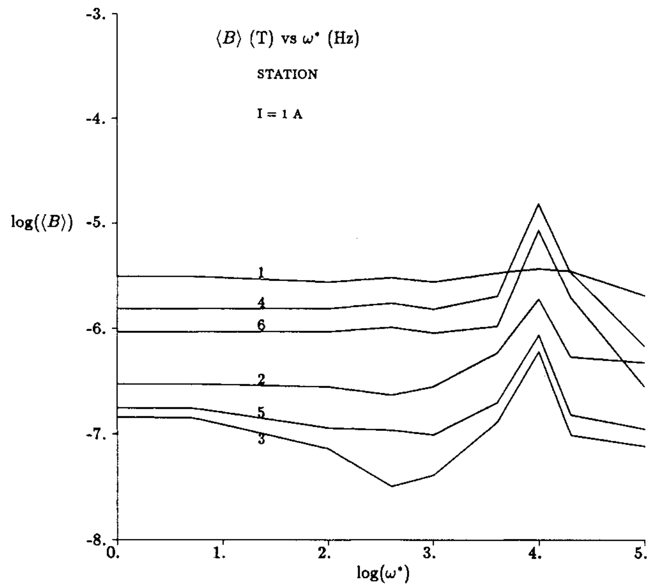

Fig. 5 Average magnitude of the magnetic field fluctuation per unit current for the same directions of current flow as in Fig. 2.

Table 6 Lower hybrid frequencies for different hydrogen, oxygen plasmas

n_{H^+}/n_e	n_{O^+}/n_e	ω_{lh_1}, Hz	ω_{lh_2}, Hz
0.1	0.9	3×10^2	8.3×10^3
1/3	2/3	1.7×10^2	1.29×10^4
0.5	0.5	1.26×10^2	1.53×10^4
2/3	1/3	1.65×10^2	1.74×10^4
0.9	0.1	5.2×10^1	2×10^4

where $Z^{H^+ - O^+}$ total impedance in $H^+ - O^+$ plasma, Z^{H^+} impedance in H^+ plasma, and Z^{O^+} impedance in O^+ plasma.

The preceding results apply only to the cases $\omega^* = 400$ Hz and $\omega^* = 20$ kHz. To consider the effect from the ac frequency, in Fig. 8 we compare total impedances for the space station as functions of ω^* in the following typical environments: 1) O^+ plasma; 2) H^+ plasma; 3) $H_2O^+ - O^+$ plasma,

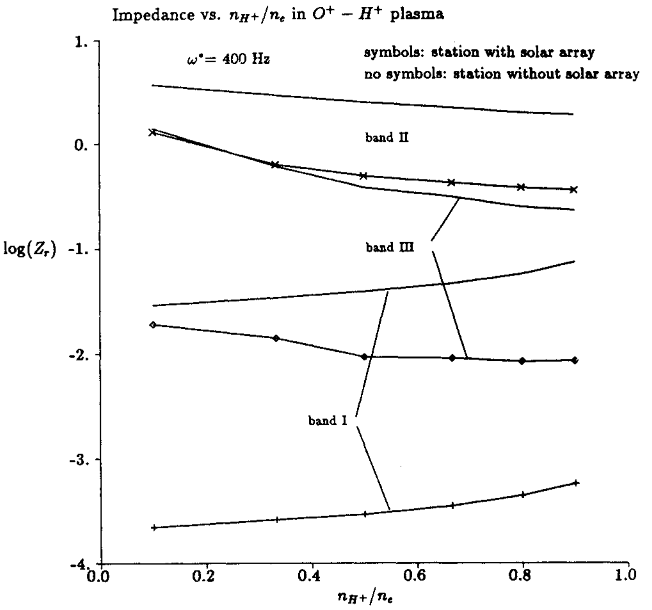

Fig. 6 Radiation impedance for a space station as a function of hydrogen to oxygen ratio at ω^* = 400 Hz.

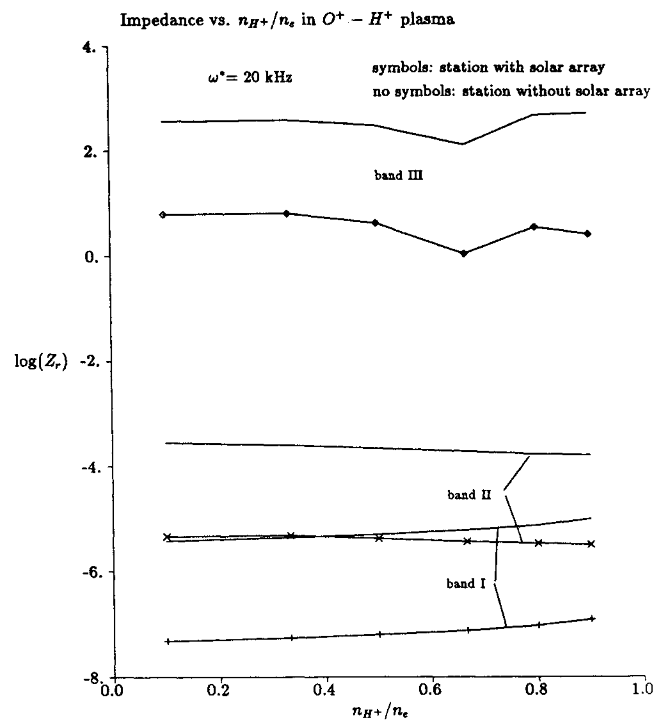

Fig. 7 Radiation impedance for a space station as a function of hydrogen to oxygen ratio at ω^* = 20 kHz.

$n_{H_2O^+}/n_e = 0.9$, $n_{O^+}/n_e = 0.1$; and 4) $H^+ - O^+$ plasma, $n_{H^+}/n_e = 0.5$, $n_{O^+}/n_e = 0.5$.

All other plasma parameters are the same. We find the following:

1) When $10^3 < \omega^* < 10^4$ Hz, the impedances in H_2O^+ and O^+ plasmas reach a maximum, whereas the impedances in H^+ and $H^+ - O^+$ plasmas have a minimum. Hence, there exists for this range

$$Z^{H_2O^+ - O^+} \sim Z^{O^+} \gg Z^{H^+} \sim Z^{O^+ - H^+}$$

2) When $\omega^* > 10^4$ Hz, the impedances in H^+ and $H^+ - O^+$ plasmas reach their maximum, and the impedances in H_2O^+

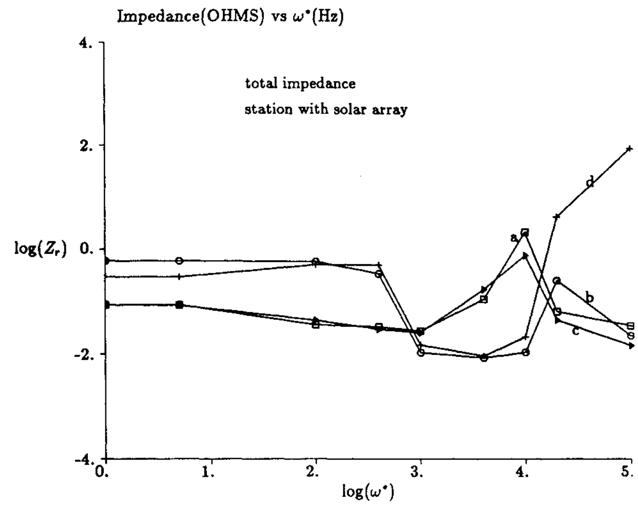

Fig. 8 Radiation impedance for a space station as a function of frequency for several different environments: a) 100% O^+; b) 100% H^+; c) 90% H_2O^+, 10% O^+; and d) 50% H^+, 50% O^+.

and O^+ plasma decay quickly. Thus for this range,

$$Z^{H^+ - O^+} \gg Z^{H^+} > Z^{H_2O^+ - O^+} \sim Z^{O^+}$$

For $\omega^* < 10^2$ Hz, we see that the radiation impedance generally decreases as the ionic mass increases. We also find from the detailed calculations that, if we insulate almost the whole solar array but leave a very tiny conducting area, we shall get a very high impedance that is hardly affected by either the environment or ω^*. In this case, the station is acting like a thin tether so that the results are similar to the tether results.[5]

In order to understand the interactions between wave and environment, we present a model that explains the preceding effects. We assume that the environment can be modeled as a medium composed of millions of damped oscillators. Those oscillators are excited if there are perturbations. Whenever there is a wave propagating through this medium, energy carried by the wave will be transferred to those oscillators to overcome the damping. Thus, radiation impedance can be represented by the total energy consumed by all the damped oscillators.

A single-ion species plasma, such as H^+ and O^+ plasma, can be modeled by a set of simple oscillators. Their characteristic frequencies would be $\omega_0^{H^+} \sim \sqrt{K/m^{H^+}}$ and $\omega_0^{O^+} \sim \sqrt{K/m^{O^+}}$ by analogy to a harmonic oscillator. A multi-ion plasma, such as a $H_2O^+ - O^+$ and $H^+ - O^+$ plasma, can be modeled by a set of coupled oscillators. There are two characteristic frequencies ω_{01} and ω_{02} for a coupled oscillator. For 90% H_2O^+-10% O^+ plasma, there is only weak coupling. Using results from classical mechanics, the characteristic frequencies would be $\omega_{01}^{H_2O^+ - O^+} \sim \omega^{O^+}(1 + \epsilon)$ and $\omega_{02}^{H_2O^+ - O^+} \sim w^{O^+}(1 - \epsilon)$, where $\epsilon \ll 1$. Thus, we have $\omega_{01}^{H_2O^+ - O^+} \sim \omega_{02}^{H_2O^+ - O^+} \sim \omega^{O^+}$. But for 50% H^+-50% O^+ plasma, strong coupling can be expected, and $\omega_{01}^{H^+ - O^+}$ and $\omega_{02}^{H^+ - O^+}$ will be very different.

If ω^* is very low, the wave propagation is determined mainly by the properties of the environment. If the perturbation of the structure matches the natural wavelength scale in this environment, then the wave will propagate very easily, and so we get a low radiation impedance. When ω^* increases, all oscillators in the medium are driven to oscillate at the same frequency. Since the oscillation of heavier oscillators consumes more energy then lighter oscillators, the propagation of a wave with a given amplitude will meet a larger "damping" force in the heavier ion plasma; hence, we get a larger impedance. When $\omega^* \sim 10^4$ Hz, it seems that $\omega_0^{O^+}$ is also approximately this value, so that a resonance is set up in an O^+ plasma. Since $\omega_{01}^{H_2O^+ - O^+} \sim \omega_{02}^{H_2O^+ - O^+} \sim \omega^{O^+}$, a resonance also happens in

$H_2O^+ - O^+$ plasma. At a resonance, the damped oscillators consume a much greater amount of energy due to resonance oscillation motion, and so a maximum value of the impedance is obtained. This is the reason that, for $10^3 < \omega^* < 10^4$ Hz, $Z^{H_2O^+ - O^+}$ and Z^{O^+} take their maximum value. If we further increase ω^*, since fewer and fewer heavier particles can follow the external driven frequency, the total "damping force" decreases. We get a smaller impedance because less energy is consumed by the damping force in a plasma composed of heavy ion species. Hence, $Z^{H_2O^+ - O^+}$ and Z^{O^+} decrease after $\omega^* > 10^4$ Hz. Since $m_{O^+}/m_{H^+} = 16$ implies $\omega_0^{H^+}/\omega_0^{O^+} \sim 4$, resonance happens in an H^+ plasma at $\omega^* \sim 4 \times 10^4$ Hz. Thus Z^{H^+} maximizes itself there. In an $H^+ - O^+$ plasma, it seems that $\omega_{01}^{H^+ - O^+}$ and $\omega_{02}^{H^+ - O^+}$ are in the range of 2×10^4 to 1×10^5 Hz; thus, we get a very big impedance in $H^+ - O^+$ plasma for $\omega^* \geq 2 \times 10^4$ Hz.

We summarize the discussion on the multi-ion species environmental effects as follows:

1) For systems with a very low alternating current (ac) frequency ($\omega^* < 100$ Hz), the major effect from a multi-ion environment will be only on the wave radiation bands.

2) For systems with a high ac frequency, since the appearance of a new kind of ion species will change the characteristic frequencies, the impedance will be affected due to resonance effects.

From the results and discussions in this section, we can obtain a few general rules to help design large active space structures such as the space station:

1) For a space structure with low ac frequency ($\omega^* < 10^2$ Hz), the effect of environment is not very significant. In order to get a lower impedance, we can increase the total current collecting area or insulate the conducting surfaces in the x_1 direction to force more current flow through the surfaces in the x_3 direction. In order to get a higher impedance, we decrease the total current collecting area or insulate the conducting surface in the x_3 direction to force more current to flow through surfaces $A_1 - A_1'$.

2) For a space structure with ac in the range $10^2 < \omega^* < 10^4$ Hz, we can still change the impedance by changing the collecting area or collecting surfaces in the aforementioned way. The impedance can also be changed by changing the surrounding environment. Usually a plasma composed of heavier ion species will result in a higher impedance, whereas a plasma composed of lighter ion species will result in a lower impedance.

3) For a space structure with ac at $\omega^* > 10^3$ Hz, since the "characteristic frequencies" for the LEO environment are usually in this same region, the impedance will take its maximum values for some frequencies in this range. For O^+ and 90% H_2O^+-10% O^+ plasma, this value is about 10^4 Hz. For space structures with ac of $\omega^* > 10^3$ Hz, we may avoid the occurrence of this maximum impedance for a fixed ω^* by modifying the environment. We note that the radiation can be eliminated by totally insulating the structure, and so no current can flow into the ionosphere.

Conclusions

The radiation impedance and power radiation have been calculated for space structures of different shapes and dimensions in several single- and multi-ion plasma environments. The radiation impedance is studied as function of ac frequency, structure, and environment. The relationships among them have been discussed.

We found that, generally, radiation impedance can be changed in the following ways:

1) Increasing total collecting area can decrease the total impedance and vice versa.

2) For a fixed geometry, a system collecting current mainly along the direction of the magnetic field possesses the minimum impedance, whereas the system collecting current mainly along the direction of motion possesses the maximum impedance.

3) For structures with ac currents below 100 Hz, generally heavy ions give lower impedances than light ions. For structures with ac current such that $\omega^* = 10^3$–10^4 Hz, those in a plasma of lighter average ion mass have smaller impedance, and those in a plasma of heavier average ion mass have larger impedance. For structures with ac current such that $\omega^* > 10^4$ Hz, usually those in a plasma of heavy and light ions have a much larger impedance than those in plasma of just one kind of ion.

4) For structures almost entirely insulated, the impedance can be very high. The higher the impedance, the weaker it depends on ω^*, structure, or environment.

The numerical and physical results obtained here can be applied directly to the design of space stations. The radiation impedance can be changed by modifying the dimension, shape, or current collecting surface of a structure as discussed. If the structure is fixed, creating an artificial surrounding environment can also change the impedance.

Acknowledgments

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